Applying Ideas from Homogeneity Analysis to Visualize Similarity Data

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Outline of the presentation

Homogeneity analysis

VOS

Empirical comparison between VOS, MDS, and DAM

Application to a large data set

Conclusions

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- A method for visualizing the relations between categories and objects

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Objective

- To provide a low-dimensional visualization in which categories and objects are located in such a way that each category is the centroid of all objects that score on the category

Constrained optimization problem

Objective function

$$\sigma(\mathbf{X}, \mathbf{Y}; \mathbf{G}) = \sum_{i<j} g_{ij} \|\mathbf{x}_i - \mathbf{y}_j\|^2$$

where $\mathbf{x}_i$ and $\mathbf{y}_j$ denote, respectively, the location of object i and category j and $g_{ij} \in \{0, 1\}$ denotes whether or not object i scores on category j

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Constraints

$$\mathbf{X}'\mathbf{X} = n\mathbf{I}$$
$$\mathbf{1}'\mathbf{X} = 0$$

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- An abbreviation for *visualization of similarities*
- A new method for visualizing similarities between objects, based on ideas similar to homogeneity analysis

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Objective

- To provide a low-dimensional visualization in which objects are located in such a way that the distance between any pair of objects reflects their similarity as accurately as possible

Mathematical notation

Input

- n: number of objects
- m: number of dimensions of the solution
- $\mathbf{S} = (s_{ij})$: $n \times n$ similarity matrix (measured on a ratio scale)

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Output

- $\mathbf{X}$: $n \times m$ coordinate matrix
  $\mathbf{x}_i = (x_{i1}, \ldots, x_{im})$ contains the coordinates of object i

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$$\mathbf{X}'\mathbf{X} = n\mathbf{I}$$
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- VOS:

$$\sum_{i<j} \|\mathbf{x}_i - \mathbf{x}_j\| = n(n-1)$$

Motivation for the objective function

- Ideal coordinates of object i

$$c_i(\mathbf{X}, \mathbf{S}) = \frac{\sum_{j \neq i} s_{ij} \mathbf{x}_j}{\sum_{j \neq i} s_{ij}}$$

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- Objective function for object i when the coordinates of all other objects are fixed

$$E_i(\mathbf{x}_i; \mathbf{X}, \mathbf{S}) = \sum_{j \neq i} s_{ij} \|\mathbf{x}_i - \mathbf{x}_j\|^2$$

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- VOS seems to have the tendency to locate objects close to their ideal coordinates

Relationship between VOS and MDS

Objective function of weighted ratio MDS

$$\sigma(\mathbf{X}; \mathbf{D}, \mathbf{W}) = \sum_{i<j} w_{ij}(d_{ij} - \|\mathbf{x}_i - \mathbf{x}_j\|)^2$$

where d_{ij} denotes the dissimilarity between objects i and j and w_{ij} denotes the weight of objects i and j

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Let $s_{ij} > 0$ for all i and j $(i \neq j)$.

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Let similarities be transformed into dissimilarities using $d_{ij} = s_{ij}^{-1}$ ($i \neq j$).

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VOS and weighted ratio MDS are then equivalent in the sense that solutions from these methods differ only by a multiplicative constant.

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Empirical comparison between VOS, MDS, and DAM (1)

Real-world bibliometric data sets

- Cocitation data of 376 journals in the field of economics and management (obtained from CWTS, Leiden University)
- Co-occurrence data on 332 concepts from the field of computational intelligence (obtained from Elsevier Scopus)
- In each data set around 75% of the similarities equal zero

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Methods

- VOS
- Unweighted ordinal MDS (SPSS PROXSCAL)
- Distance association model (DAM; De Rooij and Heijser, 2005)

Empirical comparison between VOS, MDS, and DAM (2)

VOS/MDS

- Co-occurrences/cocitations normalized using

$$s_{ij} = \frac{c_{ij}}{\left(\sum_k c_{kj}\right)\left(\sum_k c_{ik}\right)}$$

where c_{ij} denotes the co-occurrence/cocitation of objects i and j ($c_{ii} = 0$)

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DAM

- One-mode DAM with symmetric associations is used
- Row and column margins are assumed equal
- Distances are transformed into associations using the Gaussian function
- Likelihood under independent Poisson sampling is maximized

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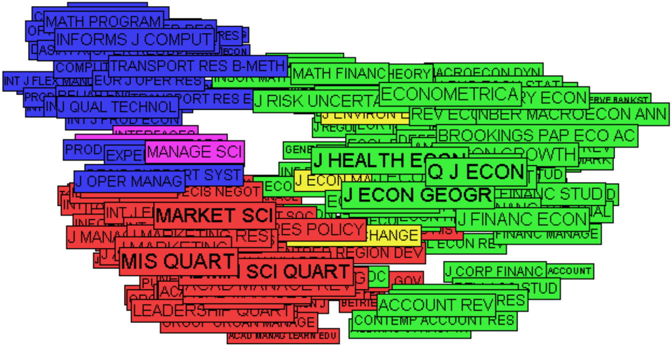
Journal cocitation map constructed using VOS



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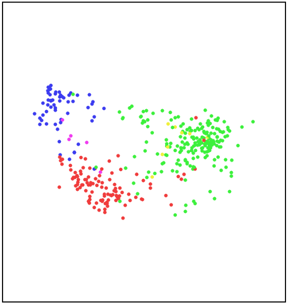
Journal cocitation map constructed using MDS

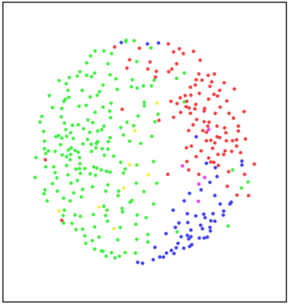
Journal cocitation map constructed using DAM

Comparison of journal cocitation maps

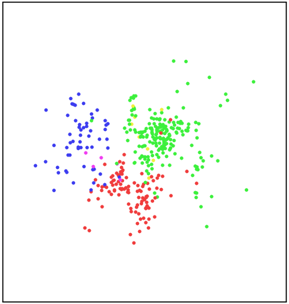
VOS



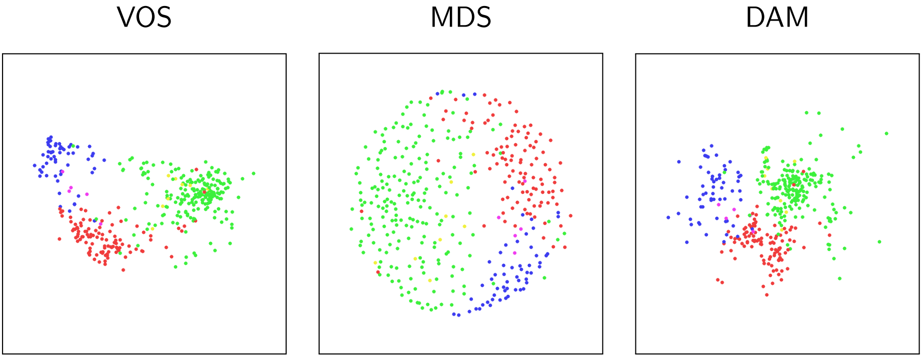
MDS



DAM



Comparison of journal cocitation maps



- MDS locates all objects roughly uniformly distributed within a circle

Comparison of journal cocitation maps

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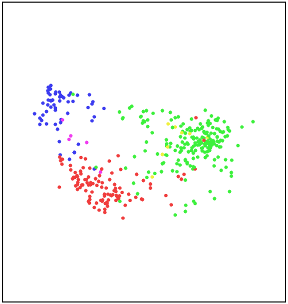
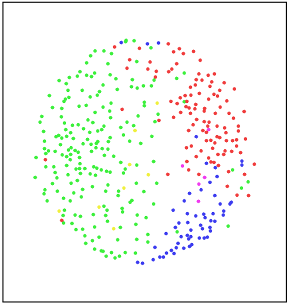
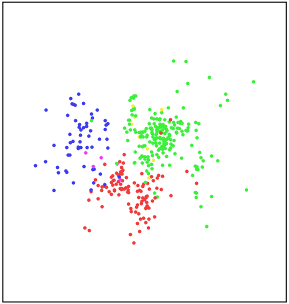


MDS



DAM



- MDS locates all objects roughly uniformly distributed within a circle
- VOS better separates the fields of economics, management, and operations research from each other than MDS and DAM

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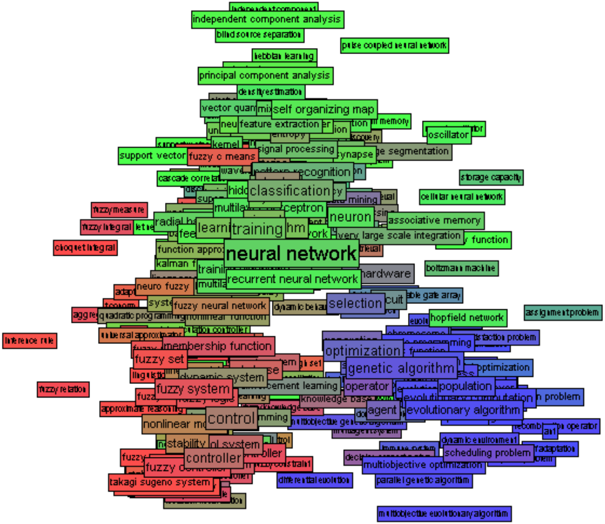
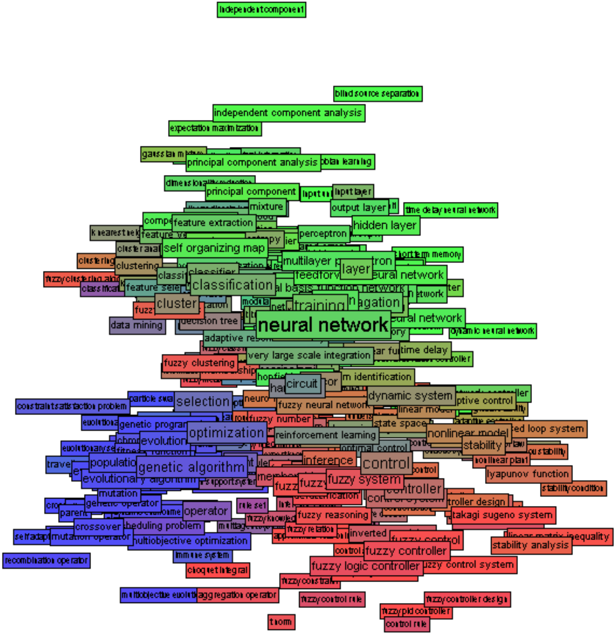
Concept co-occurrence map constructed using VOS



Concept co-occurrence map constructed using MDS

Concept co-occurrence map constructed using DAM



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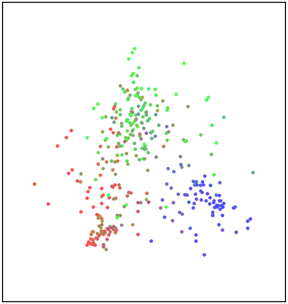
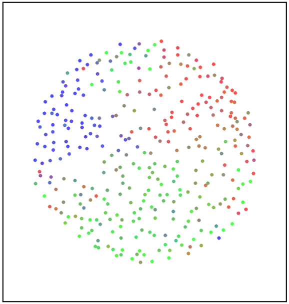
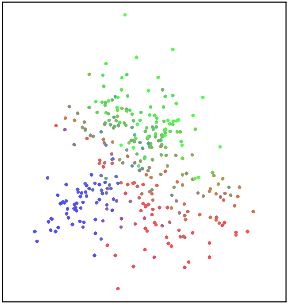


MDS



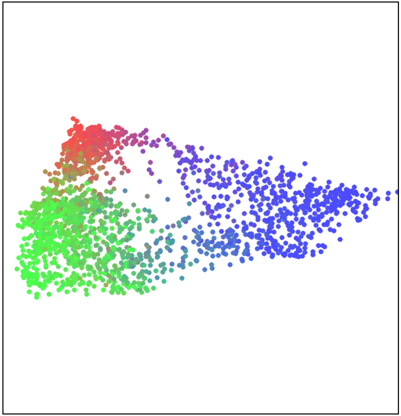
DAM

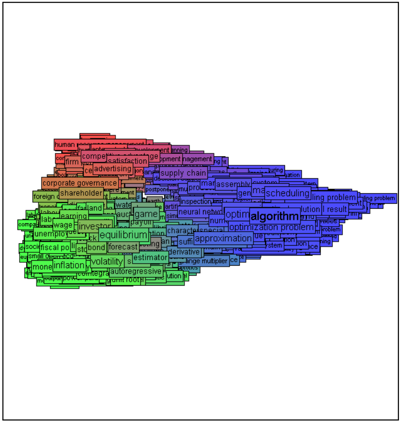


Outline of the presentation

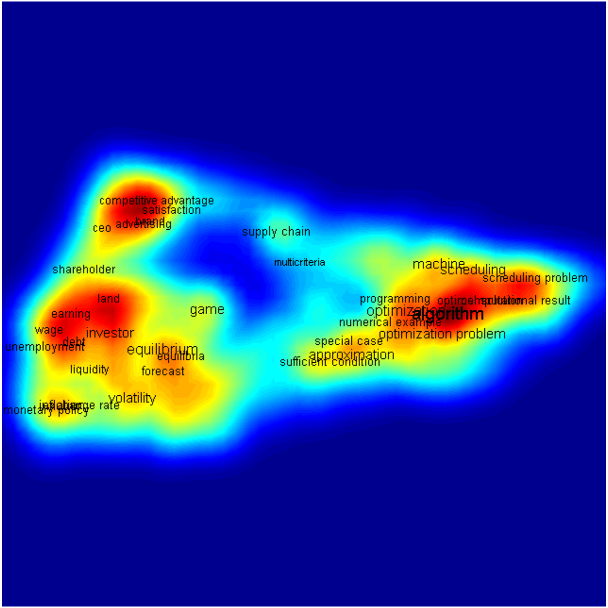
Application to a large data set

- Co-occurrence data on 2000 concepts from the field of economics and management (obtained from CWTS, Leiden University)
- Around 89% of the similarities equal zero





Concept density map



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- Unweighted ordinal MDS locates objects roughly uniformly distributed within a circle
- Solutions from VOS show better separated clusters than solutions from MDS and DAM

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Future research

- The effect on homogeneity analysis of using the VOS constraint

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More information on VOS

References to our papers and a computer implementation of VOS are available at:

`http://www.neesjanvaneck.nl/vos/`

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